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FINAL REPORT
PROJECT NO. B-185

MODES OF VIBRATION OF PIEZOELECTRIC QUARTZ CRYSTALS

By

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The purpose of this research is a better understanding of the modes of vibration of quartz piezoelectric crystals. Existing theory is inadequate to represent the limited number of systematic measurements available. The program is planned to advance the theory and to make the needed measurements for confirmation of the adequacy of the approximations necessary in arriving at a manageable theoretical simplification.

Background

The program for the year is an extension of the work done previously under the sponsorship of the U. S. Army Signal Research and Development Laboratories, Contract No. DA-36-039 SC-78905, Georgia Tech Project A-402.

A carefully prepared rectangular AT-cut crystal with third overtone frequency of 3022 kc/sec was selected for the measurements. The dimensions were 24.560 mm x_o direction, 1.650 mm in y_o, and 27.004 mm in z_o. The fundamental and third overtone frequencies depend primarily on the y_o thickness for the thickness shear mode of vibration excited by electrodes covering the faces perpendicular to this dimension. The frequency of the flexure modes also excited in this arrangement depend primarily on the x_o dimension. The experimental approach is, therefore, to record the spectrum of responses for the initial x_o dimension over a chosen frequency interval. The x_o dimension is then reduced a few microns and a new spectrum recorded. The recording equipment has been described.[1]

From a series of such spectra a mode chart can be constructed. The mode chart is a plot of the x_o dimension as abscissa and the frequency as ordnate.

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[1] Final Report, Projects A-402-11, -12, and -13, Contract No. DA-36-039 SC-78905, Quartz Crystal Studies and Measurements, 1 March 1959 to 30 June 1960

Prior to the current work, measurements in the vicinity of the third overtone were completed for the reduction of the x_o dimension from 24.56 mm to 23.55 mm, and a mode chart completed.

Identification of the mode of vibration responsible for a particular response is facilitated by a record of the polarization along one or more lines across the surface of the crystal when it is excited at the corresponding frequency. The signal generated in a probe 0.4 mm in diameter flush with the surface of one of the electrodes is recorded as the crystal, elevated 0.05 mm from the electrode, is moved slowly by. The equipment and the interpretation procedure has been described.[1]

Research Accomplished

Experimental. At the beginning of the year the equipment for recording the spectrum was rebuilt. Previously only the frequencies in the vicinity of the third overtone were recorded. Since the new theory should be tested at frequencies in the vicinity of the fundamental response, provision was made for recording the responses in this band as well. Two tracking tank circuits were therefore prepared, one for the band 2700 kc to 3300 kc and the other for the band 900 kc to 1100 kc. The spectrum recording equipment has been described in some detail.[1] One tank circuit controls the exciting frequency. A second tank circuit, carefully aligned to track the first, is inductively coupled to the first. The crystal electrodes are connected across all or part of the coil of this tank circuit.

At a response frequency remote from the thickness shear mode vibrations, the motional resistance of the crystal is high. The signal from the detector across the tank circuit is at maximum, and the deflection of the recording galvanometer is adjusted accordingly. The ganged tuning condensers of the tank circuits are motor driven so that the frequency is swept through the band in about six minutes. At this sweep rate the photographic resolution of the record (about 0.2 mm) rather than the 1200 c response of the galvanometer limits the resolution.

Each spectrum is calibrated in frequency by independent galvanometer records on each side of the spectrum. The signal from the sweep oscillator is mixed with the combined signals from auxiliary crystal oscillators of 10 kc, 100 kc, and either 1000 or 3000 kc (depending on the band) to provide the calibration signal.

During the year the x_o dimension of the crystal was reduced from 23.55 mm to 20.95 mm in 270 steps averaging 9.6 microns. The mode charts were plotted on 10 x 10 to the inch cross-section paper. Figures 1 and 2 are photographically reduced copies. The strength of the response (in inverse relation to the motional resistance) is indicated by the size of the dot plotted.

The band in the vicinity of the fundamental, Figure 1, indicates clearly the trend of the frequency of the various modes with change in the x_o dimension. The interaction of some of the modes is shown.

The mode chart for the third overtone band, Figure 2, indicates the stronger responses over the range of x_o dimension explored. The complexity of the chart is evident in the range 23.4 to 23.0 mm, where many of the weaker responses have been transferred to the mode chart.

In addition to the data for the mode charts, the motional resistance has been measured and the surface polarization pattern has been recorded for both the fundamental and third overtone responses at each reduction of the crystal.

The experimental work was carried out by Dr. M. Aruga under the guidance of Dr. I. Koga.

Theoretical. An early investigation[2] showed that the frequencies of the piezoelectrically-excited thickness vibrations of a plate of limited size are closely approximated by the frequencies computed for a plate of infinite extent. More recently,[3,4] computations of frequencies for AT-cut quartz plates having finite length-to-thickness ratios have displayed, both qualitatively and quantitatively, the intricate spectrum observed in the laboratory.

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[2] I. Koga, Physics 3, 70 (1932).

[3] I. Koga and H. Fukuyo, J. Inst. Elec. Comm. Engrs. Japan 36, 59 (1953) in Japanese) and I. Koga, J. E. Rhodes and W. B. Wrigley, Quarterly Report No. 1, Projects A-402-1, -2, and -3, Contract No. DA-36-039 SC-78910, 1 August to 1 November 1958.

[4] R. D. Mindlin, J. Appl. Phys. 22, 316 (1951).

A further extension[5] of the theory to accommodate the piezoelectric effect was subsequently accomplished, and included the interaction between the elastic and electric fields in the vibrating crystal.

Procedures heretofore presented in the literature have involved dealing directly with appropriate partial differential equations of motion for the rectangular crystal plate. A different technique which is based on a variational approach and which allows for considerable refinement in the mathematical analysis has been utilized throughout the current investigation. The theoretical results are found to agree more closely with experiment than do those presented in previous works. A brief outline of the mathematical development follows.

Hamilton's Principle when applied to a vibrating rectangular quartz crystal plate involves the strain-energy function. In addition to the mechanical energy of deformation, the electrical energy can be expressed in terms of appropriate stresses and strains within the crystal. Linear stress-strain relations are next introduced. Simplification is then achieved by assuming that deformation is due primarily to shear and flexure in planes normal to the z-axis.* The simplified strain-energy function obtained in this way is finally introduced into the mathematical expression of Hamilton's Principle.

The displacements of any point in the crystal plate are next characterized in terms of arbitrary functions of one variable. This description is based on the known results for an infinite plate and on intuitive insight regarding the adjustment which must be introduced to treat a finite plate. Integration and variation as indicated by Hamilton's Principle now are performed. The result is a set of ordinary differential equations together with concomitant boundary conditions for the free surfaces of the crystal. Solutions to the differential equations are introduced into the boundary-value relations to yield a set of linear algebraic equations for the arbitrary constants. The existence of non-trivial solutions for this latter set requires that the secular determinant vanish.

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[5] R. D. Mindlin, J. Appl. Phys. 23, 83 (1952).

* See Ref. 1 for definition

The eigenfrequencies corresponding to any specified ratio of length to thickness follow.

A program has been written for the Burrows 220 Calculator of the Rich Electronic Computer Center at Georgia Tech for the computation of the eigenfrequencies over the measured range in the ratio of dimensions. Desk calculations serve as a check on the computer results. Elastic, piezoelectric, and dielectric stiffness constants appropriate to the present calculations have been determined with the aid of experimental values reported in a previous publication.[6] Results calculated to date are in close agreement with prior measurements and will be compared later with the measurements described.

The theory has been developed by Dr. I. Koga and Dr. G. C. Knollman.

Proposed Continuation

A paper to be submitted to the Physical Review summarizing the theoretical work is complete except for the needed final comparison of computed frequencies with the measurements. This comparison awaits the final check of the computer program and the running of the results. Completion of the manuscript in about a month appears feasible.

The limited range of frequencies thus far measured provided an adequate check of the theory over a limited range of values. A more sensitive evaluation will be provided by comparison with a wider range of frequency measurements. The continuation of the experimental program, therefore, will extend the measurements to cover the band 1,100 kc to 100 kc as well as measurement of the band 3,300 to 2,700 kc for smaller values of x_o.

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[6] I. Koga, Phys. Rev. 109, 1467 (1958).

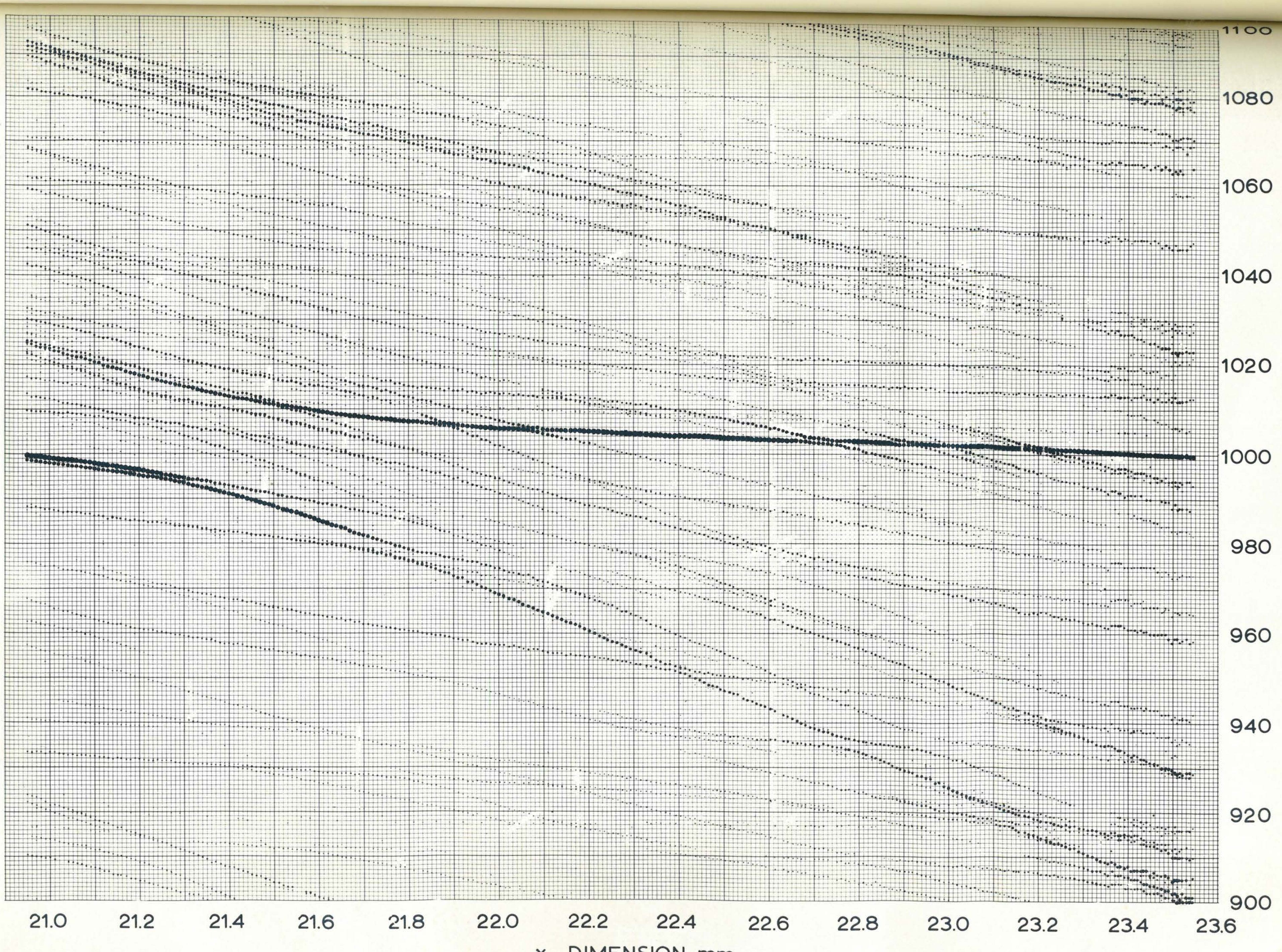

Figure 1. Responses in the Vicinity of the Fundamental

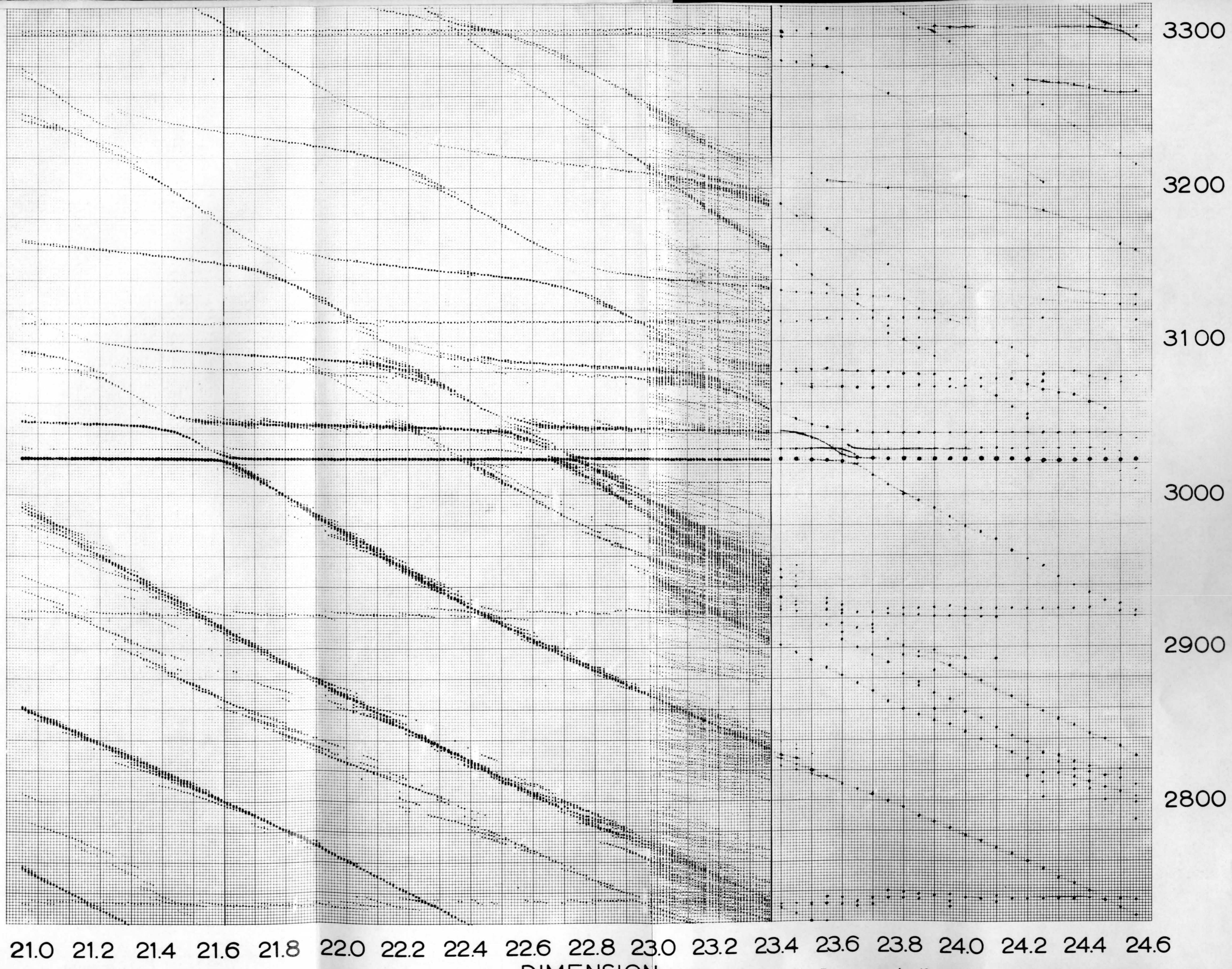

Figure 2. Responses in the Vicinity of the Third Overtone